havior of moderate- to high-aspect-ratio balanced laminated lifting surfaces as a function of ply angles. The numerical results indicate the strong influence of CVM inclusion in aeroelastic instability computations. With the inclusion of CVM, a better convergence in mode shapes is obtained regardless of the aspect-ratio effects. For moderate-aspect-ratio lifting surfaces the inclusion of CVM switches the flutter mode from second bending to first torsion, or from first torsion to second bending as a function of laminate orientation angles. At high-aspect-ratio lifting surfaces the inclusion of CVM affects the flutter speed but does not influence the flutter mode type. In divergence the inclusion of CVM becomes more critical within certain ply angles for lifting surfaces of aspect ratios larger than 6. For high-aspect-ratio lifting surfaces, a laminate design with positive top and bottom angles is recommended.

The results presented would be very helpful in design, particularly at the initial design phase to size the lifting surface to achieve mission velocities of the flight vehicle to be free of aeroelastic instabilities.

References

[1]Shirk, M., Hertz, T. J., and Weisshaar, T. A., "Aeroelastic Tailoring-Theory, Practice and Promise," *Journal of Aircraft*, Vol. 23, No. 1, 1986, pp. 6–18.

[2]Hollowell, S. J., and Dugundji, J., "Aeroelastic Flutter and Divergence of Stiffness Coupled, Graphite/Epoxy Cantilevered Plates," *Journal of Aircraft*, Vol. 21, No. 1, 1982, pp. 69–76.

[3]Oguz, S., "Aeroelastic Tailoring of a Composite Wing for Flutter and Divergence Control," M.S. Thesis, Univ. of California, Davis, CA, 1995.

[4]Spielberg, I. N., "The Two Dimensional Incompressible Aerodynamic Coefficients for Oscillatory Changes in Airfoil Camber," *Journal of Aeronautical Sciences*, Vol. 1, No. 6, 1953, pp. 432–436.

[5]Rao, S. S., *Mechanical Vibrations*, Addison–Wesley, New York, 1990, pp. 125–135.

Multiple Attractors in Inertia-Coupled Velocity-Vector Roll Maneuvers of Airplanes

Anirudh Modi* and N. Ananthkrishnan†
Indian Institute of Technology,
Bombay, Mumbai 400076, India

Nomenclature

I_x, I_y, I_z = roll, pitch, and yaw inertia, respectively
i_1, i_2, i_3 = cyclically $(I_z - I_y)/I_x$, etc.
l, m, n = roll, pitch, and yaw moment coefficients, respectively
p, q, r = roll, pitch, and yaw rates, respectively
y, z = side and normal force coefficients, respectively
α, β = angle of attack and sideslip, respectively
δa, δe, δr = aileron, elevator, and rudder deflection, respectively

Subscripts
α, p, . . . = stability derivative with respect to α, p, . . .

Superscripts
$\hat{}$ = division by either i_1, i_2, or i_3
$'$ = transpose

I. Introduction

INERTIA-coupled rapid roll maneuvers of aircraft involve nonlinear dynamics that tend to result in excessive sideslip and undesirable pitching motion, and that need to be avoided to prevent instability and the loss of pilot control.[1] In a roll maneuver initiated by an aileron deflection, this calls for use of the rudder to counter the sideslip, and elevator deflection to control the pitching motion. However, the nonlinear nature of the problem makes it difficult to prescribe the required rudder and elevator deflections for a given aileron deflection or roll rate demand.

Significant progress in the solution of the problem of inertia-coupled rapid rolls could be achieved only after the instability was identified with a jump phenomenon. Schy and Hannah[2] showed that large sideslips and pitch rates are created when the system jumps at a saddle-node bifurcation point from one attractor (stable solution) to another with typically large values of roll rate, pitch rate, and sideslip. They used the customary fifth-order pseudosteady-state (PSS) approximation[3] with a linear aerodynamic model, and aileron deflection as the parameter. In the absence of a nonlinear aerodynamic model in their analysis, the jump phenomenon could be attributed solely to the effect of the nonlinear inertia coupling terms. Their study was extended to include nonlinear aerodynamic effects by Young et al.[4] In recent years, continuation methods have been used by Jahnke and Culick[5] to evaluate jump in rapid roll maneuvers for the F-14 with a nonlinear aerodynamic model.

On the basis of Schy and Hannah's work,[2] the problem of limiting sideslip and pitch rate in inertia-coupled roll maneuvers could be restated as the equivalent problem of avoiding the saddle-node bifurcation and multiple attractors in the PSS formulation. Carroll and Mehra[6] described a nonlinear variation of the rudder deflection as a function of the aileron deflection, called an aileron–rudder interconnect (ARI), that would avoid jump. Ananthkrishnan and Sudhakar[7] devised a linear ARI strategy to avoid the saddle-node bifurcation by using the idea of tuning a perturbation parameter across a transcritical bifurcation. Inertia-coupled roll maneuvers with a zero-sideslip constraint, called coordinated rolls, were studied by Ananthkrishnan and Sudhakar.[8] Coordinated rolling was seen to avoid multiple attractors, and thus prevent jump, but this required a nonlinear ARI relationship.

Velocity-vector roll (VVR) maneuvers, where the angular velocity vector coincides with the flight path (linear velocity vector), have been considered by designers as a possible strategy to limit sideslip and pitch rate in rapid roll maneuvers of aircraft.[9] The aim of this Note is to investigate whether a VVR strategy eliminates multiple attractors and prevents jump in rapid roll maneuvers.

II. Pseudosteady-State Analysis

The fifth-order PSS equations in the five variables (p, q, r, β, and α) are solved with the linear aerodynamic model of aircraft B of Schy and Hannah.[2] The use of a linear aerodynamic model ensures that the nonlinear behavior is only a result of the inertia coupling nonlinearities. The inclusion of nonlinear aerodynamic terms to this model is expected to provide better quantitative values at large angles of attack and sideslip, while maintaining an unchanged qualitative picture. In particular, for the limited aim of this study, the presence of multiple attractors is expected to be unaffected by the nonlinear aerodynamics. In fact, it is noticed that the numerical values for jump onset obtained with the linear aerodynamic model are fairly accurate as long as the saddle-node bifurcation point occurs for small angles of attack and sideslip.

Received June 19, 1997; revision received Nov. 11, 1997; accepted for publication March 27, 1998.

*Undergraduate Student, Department of Aerospace Engineering; currently Graduate Student, Aerospace Engineering, Pennsylvania State University, University Park, PA 16802.

†Assistant Professor, Department of Aerospace Engineering.

PSS solutions for the preceding model are obtained by solving

$$(pI - M)x = c + pb \tag{1}$$

$$l_\beta \beta + l_p p + l_q q - i_1 qr + l_{\delta a}\delta a + l_{\delta r}\delta r = 0 \tag{2}$$

where I is the identity matrix

$$M = \begin{bmatrix} 0 & \hat{n}_r & \hat{n}_\beta & 0 \\ -\hat{m}_q & 0 & 0 & -\hat{m}_\alpha \\ 1 & 0 & 0 & z_\alpha \\ 0 & 1 & -y_\beta & 0 \end{bmatrix}$$

and $x = (q, r, \beta, \alpha)'$; $c = (\hat{n}_{\delta a}\delta a + \hat{n}_{\delta r}\delta r, -\hat{m}_{\delta e}\delta e, z_{\delta e}\delta e, -y_{\delta a}\delta a - y_{\delta r}\delta r)'$; and $b = (\hat{n}_p, 0, 0, 0)'$.

The maneuver considered is a roll initiated from a trim angle of attack corresponding to a 2-deg elevator deflection. PSS roll rate, sideslip, and pitch rate solutions for unconstrained rolling motion with $\delta e = 2$ deg, $\delta r = 0$, and for varying negative values of aileron deflection δa, computed by a continuation algorithm, are plotted in Figs. 1–3. Only solutions corresponding to a positive roll rate are shown in these figures. Figures 1–3 each show two stable solution (attractor) branches, and the jump from the low (p, β, q) branch to the high (p, β, q) branch takes place at the saddle-node bifurcation at $\delta a \approx -4$ deg. Similar jump phenomenon also occurs for the other variables (α, r), but these are not plotted here. The bifurcation point occurs for $(\alpha, \beta) \approx (-3, -8)$ deg.

III. VVR Constraint

The deviation of the angular velocity vector $\mathbf{\Omega}$ from the linear velocity vector $\mathbf{V}$ can be measured in terms of the angle θ between the two vectors, defined as[10]

$$\cos\theta = \frac{\mathbf{V}\cdot\mathbf{\Omega}}{|\mathbf{V}||\mathbf{\Omega}|} \tag{3}$$

The VVR constraint corresponds to $\theta = 0$ or 180 deg, i.e., $\cos\theta = \pm 1$. For small α, β, the constraint equation can be written as

$$(p + q\beta + r\alpha)/|\mathbf{\Omega}| = \pm 1 \tag{4}$$

PSS solutions with the VVR constraint have been obtained by solving Eqs. (1) and (2) with the rudder deflection δr computed to satisfy the constraint Eq. (4) for $\cos\theta = 1$, which corresponds to positive roll rate solutions. Elevator deflection is maintained unchanged at $\delta e = 2$ deg. The PSS solutions with VVR constraint are also plotted in Figs. 1–3, and they show two attractor branches with a jump between them at a saddle-node bifurcation. The bifurcation point at $\delta a \approx -12$ deg occurs for $(\alpha, \beta) \approx (-7, 2)$ deg. The PSS solutions with VVR constraint for $\delta a > 0$ can be similarly obtained with the constraint equation chosen to be $\cos\theta = -1$.

The nonzero PSS roll rate for $\delta a = 0$ with VVR constraint in Fig. 1 is because the maneuver considered is a rolling pitch down with a fixed elevator deflection of 2 deg. Because of the nonzero pitch rate at $\delta a = 0$, a nonzero roll rate is required such that the vector sum of the three angular velocities lies along the linear velocity vector to satisfy the VVR constraint. With increasing negative δa, the aircraft with VVR constraint jumps to small roll rate solutions at $\delta a \approx -12$ deg, in contrast to the jump to a large roll rate solution branch at $\delta a \approx -4$ deg in the unconstrained case. The jump also results in large positive sideslip in Fig. 2, and potential instability. Thus, it may be concluded that a VVR strategy does not eliminate multiple attractors and does not prevent jump in rapid roll maneuvers. Nevertheless, it is instructive to study the PSS solutions with VVR constraint with a view to understanding the reason for this failure.

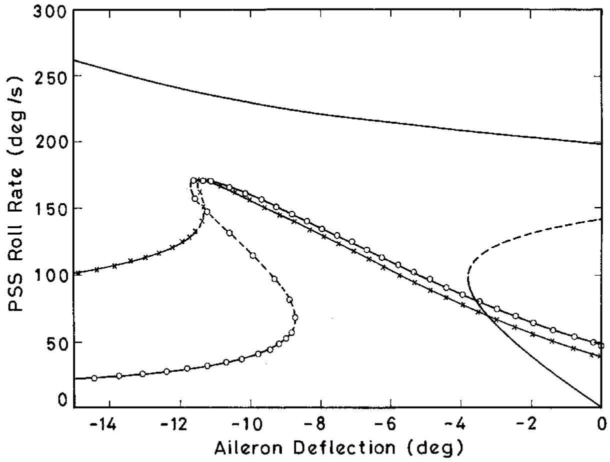

Fig. 1 PSS roll rate solutions for rolling motion with no constraint (no symbols), VVR constraint (○), and VVR, and constant pitch rate constraint (×). (All cases: ——, stable solution; - - -, unstable solution.)

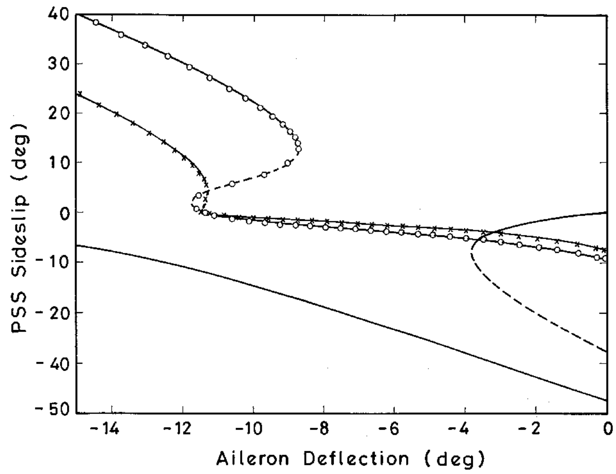

Fig. 2 PSS sideslip solutions for rolling motion with no constraint (no symbols), VVR constraint (○), and VVR and constant pitch rate constraint (×). (All cases: ——, stable solution; - - -, unstable solution.)

Figures 2 and 3 show that for a VVR maneuver, sideslip is limited below the saddle-node bifurcation point, whereas the pitch rate is nearly zero beyond the bifurcation point. This behavior can be understood by the following approximate analysis. Neglecting the $r\alpha$ term, the constraint Eq. (4) can be written as

$$(p + q\beta)/|\mathbf{\Omega}| = 1 \tag{5}$$

If $q << p$, then one can write

$$(p + q\beta)/|\mathbf{\Omega}| = 1 + (q\beta/p) = 1 \tag{6}$$

which gives the approximate constraint relation as $q\beta = 0$. This implies that the rudder acts directly to limit β on one side of the saddle-node point, and acts indirectly through the coupling terms as a control for suppressing q on the other side. This leads to the non-obvious conclusion that the VVR constraint does not necessarily translate into a constraint on the sideslip.

The analysis of the previous section suggests that if the pitch rate were independently controlled by suitably varying the elevator, then the rudder would be free to suppress the sideslip over the complete range of aileron deflections. To evaluate this possibility, PSS solutions are computed with VVR constraint as in Eq. (4) and an additional constraint requiring a constant

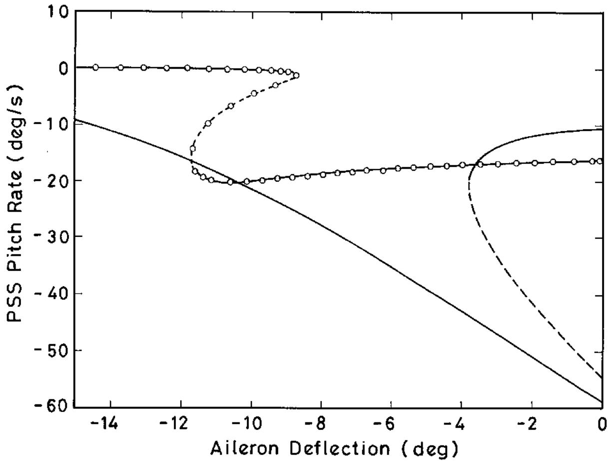

Fig. 3 PSS pitch rate solutions for rolling motion with no constraint (no symbols), and VVR constraint (○). (Both cases: ——, stable solution; - - -, unstable solution.)

pitch rate of −10.5 deg/s, which is the value of q in Fig. 3 corresponding to $p = 0$ in the unconstrained case. Computed PSS solutions for the roll rate and sideslip in this case are also shown in Figs. 1 and 2. The figures show that multiple attractors persist even with the additional pitch rate constraint. The aircraft shows a jump at $\delta a \approx -12$ deg at the saddle-node bifurcation point that occurs for $(\alpha, \beta) \approx (-4, 1)$ deg. It follows that even when the elevator is used to maintain constant pitch rate in a rapid VVR maneuver, the rudder does not suppress sideslip for large δa.

IV. Conclusions

The present study has shown that a VVR strategy does not eliminate multiple attractors and cannot prevent jump in an inertia-coupled rapid roll maneuver. In particular, for large aileron deflections in a VVR, the rudder loses influence over the sideslip and fails to suppress the sideslip.

References

[1]Pinsker, W. J. G., "The Theory and Practice of Inertia Cross-Coupling," *Aeronautical Journal*, Vol. 73, Aug. 1969, pp. 695–702.

[2]Schy, A. A., and Hannah, M. E., "Prediction of Jump Phenomena in Roll-Coupled Maneuvers of Airplanes," *Journal of Aircraft*, Vol. 14, No. 4, 1977, pp. 375–382.

[3]Hacker, T., and Oprisiu, C., "A Discussion of the Roll Coupling Problem," *Progress in Aerospace Sciences*, Vol. 15, 1974, pp. 151–180.

[4]Young, J. W., Schy, A. A., and Johnson, K. G., "Prediction of Jump Phenomena in Aircraft Maneuvers, Including Nonlinear Aerodynamic Effects," *Journal of Guidance, Control, and Dynamics*, Vol. 1, No. 1, 1978, pp. 26–31.

[5]Jahnke, C., and Culick, F., "Application of Bifurcation Theory to the High Angle of Attack Dynamics of the F-14," *Journal of Aircraft*, Vol. 31, No. 1, 1994, pp. 26–34.

[6]Carroll, J. V., and Mehra, R. K., "Bifurcation Analysis of Nonlinear Aircraft Dynamics," *Journal of Guidance, Control, and Dynamics*, Vol. 5, No. 5, 1982, pp. 529–536.

[7]Ananthkrishnan, N., and Sudhakar, K., "Prevention of Jump in Inertia-Coupled Roll Maneuvers of Aircraft," *Journal of Aircraft*, Vol. 31, No. 4, 1994, pp. 981–983.

[8]Ananthkrishnan, N., and Sudhakar, K., "Inertia-Coupled Coordinated Roll Maneuvers of Airplanes," *Journal of Aircraft*, Vol. 32, No. 4, 1996, pp. 883–884.

[9]Rajeeva Kumar, "Flight Dynamics—A Control Engineer's Perspective," *Aircraft Flight Control and Simulation*, edited by S. Chetty and P. Madhuranath, SP 9717, National Aerospace Labs., Bangalore, India, 1997, pp. 1–30, Chap. 6.

[10]Ogburn, M. E., Nguyen, L. T., and Hoffler, K. D., "Modeling of Large-Amplitude High-Angle-of-Attack Maneuvers," *Proceedings of the AIAA Atmospheric Flight Mechanics Conference* (Minneapolis, MN), AIAA, Washington, DC, 1988, pp. 244–253.

Mean Flow Refraction Corrections for the Kirchhoff Method

Anthony R. Pilon*
Pennsylvania State University, University Park, Pennsylvania 16801
and
Anastasios S. Lyrintzis†
Purdue University, West Lafayette, Indiana 47907

Introduction

NOISE generated by supersonic and subsonic jets is important for civil and military aircraft. The success of the High-Speed Civil Transport (HSCT) and other aircraft depends on a substantial reduction of the radiated jet exhaust noise. Thus it is necessary to have accurate jet noise prediction methods so that future aircraft designs can be assessed. One attractive prediction technique is the Kirchhoff method. The Kirchhoff method consists of the calculation of the nonlinear near field, usually numerically. The far-field acoustics are then determined through Kirchhoff's integral formulation, evaluated on a control surface surrounding the nonlinear field.

Kirchhoff's integral equation has recently become popular as a tool for numerical acoustic prediction.[1] Methods based on this integral relation are attractive because they utilize surface integrals, not the volume integrals found in acoustic analogy methods, over a source region to determine far-field acoustics. Additionally, Kirchhoff methods do not suffer the dissipation and dispersion errors found when the mid-field and far-field sounds are directly calculated with an algorithm similar to those used in computational fluid dynamics (CFD) studies.

The Kirchhoff method has been used successfully in the prediction of jet noise by several researchers.[2–4] Shih et al.[5] showed that the Kirchhoff method can predict results nearly identical to those obtained with a direct calculation method, with a substantial savings in CPU time. However, there are some difficulties involved with using the Kirchhoff method, and related methods, for jet aeroacoustic problems. For an accurate prediction the Kirchhoff control surface must completely enclose the aerodynamic source region. This is often difficult or impossible to accomplish with the source regions found in jet acoustic problems. The validity of predictions is also dependent on the control surface being placed in a region where the linear wave equation is valid. Difficulties meeting this criterion frequently arise in jet acoustic studies. Additionally, the existence of steady mean flow gradients outside the Kirchhoff surface will cause refraction of the propagating sound. Failure to account for this refraction will also lead to errors when the observer location is near the jet axis.

This Note outlines the development of corrections to the Kirchhoff method to account for the difficulties caused by mean flow refraction. The corrections are based on geometric acoustic principles, with the steady mean flow approximated as an axisymmetric parallel shear flow. Sample calculations are presented that show the corrections to predict a zone of silence in qualitative agreement with experimental observations. More details can be found in Ref. 6.

Presented as Paper 97-1654 at the AIAA/CEAS 3rd Aeroacoustics Conference, Atlanta, GA, May 12–14, 1997; received June 16, 1997; revision received April 10, 1998; accepted for publication April 15, 1998.

*Research Associate, Department of Aerospace Engineering. Member AIAA.

†Associate Professor, School of Aeronautics and Astronautics. Associate Fellow AIAA.